

# Scalar Field Cosmology II: Superfluidity and Quantum Turbulence

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## Abstract

We generalize the big-bang model in a previous paper by extending the real vacuum scalar field to a complex vacuum scalar field, in the context of Einstein's equation with Robertson-Walker metric. From the phase dynamics of the complex scalar field emerges superfluidity, vorticity, and quantum turbulence, which corresponds to a fractal vortex tangle. We propose that such a tangle fills the early universe, in which matter was created through the reconnection of vortex lines, a process necessary for its maintenance. This is implemented in a set of closed cosmological equations that describes the cosmic expansion driven by the scalar field on the one hand, and the vortex-matter dynamics on the other. We show how these two aspects decouple from each other, due to a vast difference in energy scales. In this model, the lifetime of the vortex tangle gives a reasonable quantitative account of the era of cosmic inflation. The model is not valid beyond the inflation era, but has qualitative predictions for the later universe on the basis of superfluidity and vorticity. These include the galactic voids, "dark mass", "non-thermal filaments", and cosmic jets.

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## I. INTRODUCTION AND SUMMARY

In paper I of this series [1], we formulated and solved an initial-value problem for big-bang cosmology, based on Einstein's equation in Robertson-Walker (RW) metric, with a Halpern-Huang (HH) [2] quantum scalar field as source of gravity. What distinguishes a quantum field from a classical one is the existence of virtual processes, which require a high-energy cutoff  $\Lambda$ , which set the only scale in the field theory. Since there is only one length scale  $a$  in the early universe, as set by the RW metric, one must put  $\Lambda = 1/a$ .

According to renormalization theory the field potential is a function of  $\Lambda$ . The potential in the HH theory has the property of asymptotic freedom, i.e., that it vanishes in the limit  $\Lambda \rightarrow \infty$ , or  $a \rightarrow 0$ . This is a necessary property for a consistent initial-value problem. In addition to asymptotic freedom, the HH potential also exhibits spontaneous symmetry breaking, i.e., it has a minimum at a nonzero value of the field. This enables us to use a semi-classical approach, neglect quantum fluctuations and treat the vacuum field classically.

The relation  $\Lambda = 1/a$  creates a dynamic feedback loop between quantum field and gravity that is an essential feature of the model, leading to the result that the Hubble parameter decays in time according to a power law:  $H \sim t^{-p}$  ( $0 < p < 1$ ). This implies  $a \sim \exp t^{1-p}$ , which signifies accelerated expansion of the universe, indicating "dark energy". Simple models of dark energy invokes Einstein's cosmological constant, which can be identified with  $H^2$ . The power-law decay of the latter avoids the usual "fine-tuning" problem.

In paper I, we had considered two questions related to the problem of cosmic inflation, which is inseparable from matter creation:

- What mechanism was responsible for creating all the matter in the universe, before the cosmic expansion caused them to fall out of each other's horizon?
- How could the matter energy scale, of order 1 GeV, be decoupled from the Planck scale of  $10^{18}$  GeV that was built into Einstein's equation?

Within the original model with a real scalar field, we were not able to find answers satisfactory to us. We then extended the model to a complex scalar field, hoping to find new physics that would offer "natural" explanations. We did find them in the phenomena of superfluidity and quantum turbulence, which emerge from the spatial variation of the phase of the complex field. This paper is an account of this development.

The existence of a complex vacuum field makes the universe a superfluid with emergent vortex dynamics. Decades of research has given us a good understanding of superfluid vorticity in liquid helium [3,4] and Bose-condensed atomic gases [5]. We take advantage of what has been learned in those fields in our approach to the cosmic superfluidity. Feynman [6] advanced the idea of quantum turbulence, corresponding to the presence of a vortex tangle, and Vinen [7] has proposed a phenomenological equation governing its development. The idea of quantum turbulence has been well established, both theoretically and experimentally.

We propose that it dominates the inflation era in the early universe, and we implement this idea by incorporating Vinen's equation into a set of cosmological equations based on Einstein equation with RW metric and the HH scalar field. This generalizes the model discussed in I.

Quantum vorticity is different from classical vorticity, and in many ways simpler. The most important physical process responsible for the formation of the vortex tangle is the reconnection of vortex lines. Through this mechanism, large quantized vortex rings degrade into ever smaller rings, and eventually become the raging emulsion that is quantum turbulence. The signature of a vortex reconnection is that two cusps appear in the vortex lines formed immediately after reconnection, and they spring away instantaneously from each other, theoretically with infinite speed. This would create two opposing jets of energy. In a vortex tangle in the early universe, reconnections takes place continually, at the rate of about one per Planck time, or  $10^{-43}$ s, with each reconnection releasing the order of one unit of Planck energy, or  $10^{18}$  GeV. An amount of energy of the order of the present total energy of the universe can be released during the lifetime of the cosmic vortex tangle, when we figure in the number of reconnections per unit volume. This conceptually answers our quest for an efficient way to create matter.

It is interesting to note that solar flares on the sun's surface require the conversion of a large amount of potential to kinetic energy in a very short time, and this was apparent achieved through the reconnection of magnetic flux lines, not unlike vortex reconnections.

We suggests that all matter is created through vortex reconnections in the quantum turbulence, and that the era of cosmic inflation is the lifetime of the vortex tangle, whose rise and fall is described phenomenologically by Vinen's equation. It is possible to choose model parameters such that the lifetime of the vortex tangle is of order  $10^{-26}$ s, during which time the radius of the universe increases by a factor of order  $10^{27}$ , and the total amount of matter created was equal to what we have now, of the order of  $10^{22}$  suns.

An important feature of our cosmological equations is the decoupling between the matter energy scale and the Planck scale. It occurs because, due to the structure of Vinen's equation, the equations can be split into two sets, one governing the scalar field and cosmic expansion, the other describing the vortex-matter system, with a link whose strength depends on the ratio (matter energy scale)/(Planck scale) $\sim 10^{-18}$ . Decoupling occurs dues to the extreme smallness of this number. This explains, from the viewpoint of Einstein's equation, why one

can do calculations on stellar structure without having to worry about cosmic expansion, and vice versa.

At the end of the inflation era, with the demise of the vortex tangle, our model ceases to be valid, because the universe has grown sufficiently large that density variations become important. However, with the decoupling of scales, the model prepares the stage for the standard "hot big bang theory" [8]. The legacy of our model in the post-inflation universe is superfluidity and vorticity, and these have observable manifestations, among which are galactic voids, the "dark mass", the "non-thermal filaments", and cosmic jets, which are discussed later in this paper.

In summary, this model offers explanations of diverse cosmic phenomena from a unified picture, namely a cosmic superfluid arising from a vacuum complex scalar field, with a full range of vortex activities.

## II. COMPLEX SCALAR FIELD AND SUPERFLUID VORTEX DYNAMICS

A complex scalar field  $\phi(x)$  is equivalent to a two-component real field  $\{\phi_1(x), \phi_2(x)\}$ , with the relation

$$\begin{aligned}\phi &= \frac{\phi_1 + i\phi_2}{\sqrt{2}} = F e^{i\sigma} \\ \phi^* &= \frac{\phi_1 - i\phi_2}{\sqrt{2}} = F e^{-i\sigma}\end{aligned}\tag{1}$$

where we introduce the phase representation, with modulus  $F$  and phase  $\sigma$ . The classical Lagrangian density is given by

$$\mathcal{L}_\phi = -g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi - V$$

In the quantum field theory, in order to tame high-frequency virtual processes, the operator  $g^{\mu\nu} \partial_\mu \partial_\nu$  in the kinetic term is "regulated" by the introduction of a small-distance cutoff, or equivalently to a high-momentum cutoff  $\Lambda$ . As in I, we set  $\Lambda = a^{-1}(t)$ , where  $a(t)$  is the scale set by the RW metric.

We use the HH potential

$$\begin{aligned}V(\phi) &= \Lambda^4 U_b(z) \\ U_b(z) &= c a^b [M(-2 + b/2, 1, z) - 1] \\ z &= 16\pi^2 \Lambda^{-2} \phi^* \phi\end{aligned}\tag{2}$$

Its derivative can be represented in the form

$$\begin{aligned}\frac{\partial V}{\partial \phi^*} &= \Lambda^4 U'_b(z) \frac{dz}{d\phi^*} = \Lambda^4 16\pi^2 \phi U'_b(z) \\ U'_b(z) &= -c\Lambda^{-b} \left(2 - \frac{b}{2}\right) M(-1 + b/2, 2, z)\end{aligned}\quad (3)$$

The classical equation of motion of the scalar field is

$$\partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu \phi] - \sqrt{-g} \frac{\partial V}{\partial \phi^*} = 0 \quad (4)$$

In the phase representation this reads

$$\begin{aligned}\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu F) - g^{\mu\nu} F \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} \frac{\partial V}{\partial F} &= 0 \\ \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \sigma) &= 0\end{aligned}\quad (5)$$

The complex scalar field is commonly used in condensed matter physics as the order parameter for superfluidity, with the superfluid velocity defined by

$$\mathbf{v} = \nabla \sigma \quad (6)$$

This is of dimension  $(\text{length})^{-1}$ . To obtain a velocity, it is customary to multiply  $\nabla \sigma$  by a unit of vorticity  $\kappa_0 = h/m_0$ , where  $h$  is Planck's constant, and  $m_0$  is a mass parameter from the power series  $V = m_0^2 \phi^* \phi + \dots$ . For simplicity we omit this factor.

The presence of a vacuum complex scalar field makes the universe a superfluid, a salient feature of which is the quantization of vorticity. Around any closed circuit  $C$ , the phase  $\sigma$  can only change by a multiple of  $2\pi$ , since  $\phi(x)$  must be continuous. This lead to the quantization condition

$$\oint_C \mathbf{v} \cdot d\mathbf{s} = 2\pi n \quad (7)$$

where the line integral is carried around any closed curve  $C$  in space, and  $n$  is an integer. If  $n \neq 0$ , then  $C$  cannot be shrunk to zero; it encircles a directed line called the vortex line, on which  $\phi = 0$ . The vortex line cannot terminate inside the superfluid; it either forms a closed loop, or terminate on a surface. We only need consider  $n = 1$ , for higher vortices tend to be unstable and break up into lower ones, when perturbations are present.

The velocity tends to infinity at the vortex line, and the modulus  $F$  must vanish on the line to keep the energy finite. Thus, the vortex line renders the space non-simply connected,

and we can have  $\nabla \times \mathbf{v} \neq 0$ , even though  $\mathbf{v}$  is a gradient. We write

$$\nabla \times \mathbf{v} = \mathbf{j} \quad (8)$$

where  $\mathbf{j}(x)$  is the vorticity density. This is analogous to Maxwell's equation for a magnetic field due to a current in a wire shaped like the vortex line. The solution is the Biot-Savart law

$$\mathbf{v}(\mathbf{r}, t) = \int_{\mathcal{L}} \frac{(\mathbf{s} - \mathbf{r}) \times d\mathbf{s}}{|\mathbf{s} - \mathbf{r}|^3} \quad (9)$$

where  $\mathbf{s}$  is the vector position of a point on the vortex line, and the integration ranges over  $\mathcal{L}$ , the totality of all vortex lines. The integral above diverges when  $\mathbf{s} \rightarrow \mathbf{r}$ , and a cutoff is needed. In liquid helium the cutoff comes from atomicity, and in our case it comes from the field-theory cutoff  $\Lambda^{-1} = a(t)$ . This replaces the vortex line by a tube called the vortex core, whose radius should be proportional to  $a(t)$ , since that is the only length scale available. We shall continue to refer to the center of the core as the vortex line.

The field modulus  $F$  inside the vortex core is suppressed, with functional form determined by the cutoff function. We adopt the simple model that  $F$  is zero inside, and constant outside. In this picture, the scalar field can be regarded as uniform in space, except that the space is made non simply-connected, by exclusion of the vortex tube. A static vortex solution is discussed in Appendix A, and vortex dynamics is reviewed in Appendix B.

The vortex tube cannot spontaneously arise, but must be nucleated by quantum fluctuations of the scalar field. A microscopic ring-shape tube would appear by fluctuation, and grow to macroscopic dimensions under appropriate conditions. The vortex tube has an energy cost of  $\epsilon_0$  per unit length, and its presence induces superfluid flow. Thus, its energy consists of two parts:

$$\begin{aligned} \epsilon_0 &= \text{energy per unit length of vortex tube} \\ v^2 &= \text{energy density of induced superfluid flow} \end{aligned} \quad (10)$$

We work within the RW metric, which assumes spatial uniformity. To conform this requirement in the equations of motion (5), we assume that  $F$  is constant in space outside of the vortex tube, and we perform spatial averages on terms involving the phase  $\sigma$ . The

resulting equations of motion are

$$\begin{aligned}\ddot{F} &= -3H\dot{F} + F\langle\dot{\sigma}^2\rangle - F\langle|\nabla\sigma|^2\rangle - \frac{1}{2}\frac{\partial V}{\partial F} \\ \frac{d}{dt}\langle\dot{\sigma}\rangle &= -3H\langle\dot{\sigma}\rangle\end{aligned}\tag{11}$$

where  $H = \dot{a}/a$ , and  $\langle\rangle$  denotes spatial average. The space, however, is the non-simply connected region outside of vortex tubes. The energy density and pressure of the scalar field are given by

$$\begin{aligned}\rho_\phi &= \dot{F}^2 + \langle\dot{\sigma}^2\rangle + V \\ p_\phi &= \dot{F}^2 + \langle\dot{\sigma}^2\rangle - V - \frac{a}{3}\frac{\partial V}{\partial a}\end{aligned}\tag{12}$$

where the  $\partial V/\partial a$  term is explained in I. The second equation in (11) gives  $\langle\dot{\sigma}\rangle \propto a^{-3}$ , which will rapidly vanish as  $a$  increases. We assume  $\langle\dot{\sigma}^2\rangle \sim O(a^{-6})$ , and neglect it. Thus we have

$$\ddot{F} = -3H\dot{F} - F\langle v^2\rangle - \frac{1}{2}\frac{\partial V}{\partial F}\tag{13}$$

with

$$\begin{aligned}\rho_\phi &= \dot{F}^2 + V + \langle v^2\rangle \\ p_\phi &= \dot{F}^2 - V - \langle v^2\rangle - \frac{a}{3}\frac{\partial V}{\partial a}\end{aligned}\tag{14}$$

The vortex tubes created in the early universe must have a core radius proportional to  $a(t)$  of the RW metric, since that is only length scale available. This core will expand with the universe, maintaining the same fraction of the radius of the universe, and may account for the presently observed voids in the galactic distribution, as we shall discuss later.

### III. VINEN'S EQUATION

The formation of quantum turbulence in the form of a vortex tangle is discussed in Appendix B. In this model, being restricted to spatial uniformity through use of the RW metric, we describe the tangle with one variable  $\ell(t)$ , the vortex line density (average length per unit volume). This quantity obeys Vinen's phenomenological equation, which in flat space-time has the form

$$\dot{\ell} = A\ell^{3/2} - B\ell^2$$



where  $A$  and  $B$  are phenomenological parameters. The generalization to curved space-time is

$$g^{-1/2} \frac{d}{dt} (g^{1/2} \ell) = A \ell^{3/2} - B \ell^2 \quad (15)$$

which in RW metric reduces to

$$\dot{\ell} = -3H\ell + A\ell^{3/2} - B\ell^2 \quad (16)$$

The energy density of the vortex tangle is

$$\rho_v = \epsilon_0 \ell \quad (17)$$

Vinen's equation thus states

$$\dot{\rho}_v = -3H\rho_v + \alpha\rho_v^{3/2} - \beta\rho_v^2 \quad (18)$$

where  $\alpha$  and  $\beta$  are model parameters.

As explained in Appendix B, two vortex lines undergo reconnection when they approach each other to within a distance  $\delta \propto v^{-1}$ , where  $v$  is their relative speed, which is of the same order as the average speed in the superfluid. Thus, in steady-state, the average spacing between vortex lines should be  $\delta$ . On the other hand, by geometrical considerations, the average spacing should be of order  $\ell^{-1/2}$ . This gives the estimate

$$\langle v^2 \rangle = \zeta_0 \rho_v \quad (19)$$

where  $\zeta_0$  is a constant.

The parameters  $\alpha, \beta, \zeta_0$  may depend on  $a(t)$ , for they could depend on the radius of the vortex core .

#### IV. COSMOLOGICAL EQUATIONS WITH QUANTUM TURBULENCE AND MATTER CREATION

Let us review the framework for the cosmological equations, i.e., Einstein's equation with RW metric. The cosmic expansion is described by  $a(t)$ , the scale of the RW metric, and we introduce the Hubble parameter  $H = \dot{a}/a$ . The equation are (with  $4\pi G = 1$ )

$$\begin{aligned}
\dot{H} &= \frac{k}{a^2} - (p + \rho) \\
X &\equiv H^2 + \frac{k}{a^2} - \frac{2}{3}\rho = 0 \\
\dot{\rho} &= 3H(\rho + p)
\end{aligned} \tag{20}$$

where  $\rho$  and  $p$  are respectively the total energy density and pressure derived from the energy-momentum tensor  $T^{\mu\nu}$  of non-gravitational systems. The second equation, of the form  $\dot{X} = 0$ , is a constraint on initial values. The third equation is the conservation law  $T^{\mu\nu}_{;\mu} = 0$ , and it guarantees  $\dot{X} = 0$ . The inclusion of (13) and (18) will complete the dynamics and close the equations.

So far we have three independent variables: the scale of the universe  $a$ , the modulus of the vacuum scalar field  $F$ , and the energy density of the vortex tangle  $\rho_v$ . We now introduce matter, modeled as a classical perfect fluid of energy density  $\rho_m$ . Its pressure is taken to be  $p_m = w_0 \rho_m$ , where  $w_0$  is the equation-of-state parameter, with possible values  $\{-1, 0, 1/3\}$  corresponding respectively to "vacuum energy", "pressureless dust", and "radiation". The total energy density  $\rho$  and total pressure  $p$  are now given by

$$\begin{aligned}
\rho &= \rho_\phi + \rho_m + \rho_v \\
p &= p_\phi + w_0 \rho_m
\end{aligned} \tag{21}$$

The cosmological equations are then given by

$$\begin{aligned}
\dot{H} &= \frac{k}{a^2} - (\rho + p) \\
\ddot{F} &= -3H\dot{F} - F\langle v^2 \rangle - \frac{1}{2} \frac{\partial V}{\partial F} \\
\dot{\rho}_v &= -3H\rho_v + \alpha\rho_v^{3/2} - \beta\rho_v^2
\end{aligned} \tag{22}$$

with constraint and conservation equations

$$\begin{aligned}
X &\equiv H^2 + \frac{k}{a^2} - \frac{2}{3}\rho = 0 \\
\dot{X} &= 0
\end{aligned} \tag{23}$$

The equation  $\dot{X} = 0$  acts as the equation of motion for matter. To emphasize this, we can rewrite the cosmological equations in the following form:

$$\begin{aligned}
\dot{H} &= \frac{k}{a^2} - 2\dot{F}^2 + \frac{a}{3} \frac{\partial V}{\partial a} - (1 + w_0) \rho_m - \rho_v \\
\ddot{F} &= -3H\dot{F} - \zeta_0 \rho_v F - \frac{1}{2} \frac{\partial V}{\partial F} \\
\dot{\rho}_v &= -3H\rho_v + \alpha \rho_v^{3/2} - \beta \rho_v^2 \\
\dot{\rho}_m &= -3H(1 + w_0) \rho_m - \alpha \rho_v^{3/2} + \beta \rho_v^2 + \frac{dF^2}{dt} \zeta_0 \rho_v
\end{aligned} \tag{24}$$

where the last equation is a rewrite of  $\dot{X} = 0$ . The constraint on initial conditions

$$X \equiv H^2 + \frac{k}{a^2} - \frac{2}{3} \rho = 0 \tag{25}$$

is now preserved by the equations of motion.

Finally, we introduce the total energies

$$\begin{aligned}
E_v &= a^3 \rho_v \\
E_m &= a^{3(1+w_0)} \rho_m
\end{aligned} \tag{26}$$

which will absorb the kinematic terms proportional to  $3H$  in the equations. Note that  $w_0$  appears above as an "anomalous dimension" [9]. For simplicity, we put  $w_0 = 0$ , corresponding to pressureless dust. The cosmological equations plus constraint then become

$$\begin{aligned}
\dot{H} &= \frac{k}{a^2} - 2\dot{F}^2 + \frac{a}{3} \frac{\partial V}{\partial a} - \frac{1}{a^3} (E_m + E_v) \\
\ddot{F} &= -3H\dot{F} - \frac{\zeta_0}{a^3} E_v F - \frac{1}{2} \frac{\partial V}{\partial F} \\
\dot{E}_v &= s_1 E_v^{3/2} - s_2 E_v^2 \\
\dot{E}_m &= -s_1 E_v^{3/2} + s_2 E_v^2 + \frac{dF^2}{dt} \zeta_0 E_v \\
X &\equiv H^2 + \frac{k}{a^2} - \frac{2}{3} \rho = 0
\end{aligned} \tag{27}$$

where

$$\rho = \dot{F}^2 + V + \frac{1 + \zeta_0}{a^3} E_v + \frac{1}{a^3} E_m \tag{28}$$

and

$$\begin{aligned}
s_1 &= \alpha a^{-3/2} \\
s_2 &= \beta a^{-3}
\end{aligned} \tag{29}$$

This constitutes a self-consistent and self-contained initial-value problem.

## V. DECOUPLING

In our model, matter dynamics is governed by an energy scale supplied through the parameters  $s_1, s_2$ . We take it to be the nuclear scale of order 1 GeV, which is independent of the Planck scale of  $10^{18}$  GeV built into Einstein's equation. In particle theory, it emerges spontaneously from a scale-invariant QCD, through the formation of the nucleon bound state. This mechanism is called "dimensional transmutation", the simplest mathematical example of which is the occurrence of a bound state in an attractive  $\delta$ -function potential in the 2D Schrödinger equation [10].

In a later universe, the nuclear scale and the Planck scale are decoupled from each other, for we can calculate stellar structure without worrying about cosmic expansion, and vice versa. That is, the components of Einstein's equation must be separable into near-independent sets describing matter and expansion, respectively. We now show how this could come about in our cosmological equations (27).

We define a nuclear time variable  $\tau = s_1 t$ , and assume

$$\frac{\tau}{t} = s_1 = \frac{\text{Planck time scale}}{\text{Nuclear time scale}} = \frac{\text{Nuclear energy scale}}{\text{Planck energy scale}} \sim 10^{-18} \quad (30)$$

The vortex-matter equations can be rescaled to read

$$\begin{aligned} \frac{dE_v}{d\tau} &= -E_v^2 + \gamma E_v^{3/2} \\ \frac{dE_m}{d\tau} &= E_v^2 - \gamma E_v^{3/2} + \frac{\zeta_0}{s_1} \frac{dF^2}{dt} E_v \end{aligned} \quad (31)$$

where  $\gamma = s_2/s_1$ , which we assume is of order unity. In these equations, the only link to the expanding cosmos is the factor  $\zeta_0 s_1^{-1} dF^2/dt$ , which is extremely rapidly varying in terms of  $\tau$ , with time average

$$K_0(\tau) = \left\langle \frac{\zeta_0}{s_1} \frac{dF^2}{dt} \right\rangle$$

This is a very large number, of order  $1/s_1 \sim 10^{18}$ , and would dominate the right side of the second equation in (31). Thus we can replace the vortex-matter equations by

$$\begin{aligned} \frac{dE_v}{d\tau} &= -E_v^2 + \gamma E_v^{3/2} \\ \frac{dE_m}{d\tau} &= K_0(\tau) E_v \end{aligned} \quad (32)$$

These equations are now purely in nuclear time scale. The first is Vinen's equation governing the growth and decay of the vortex tangle, and the second give the rate of matter production. The Planck time scale is retained only in the parameter  $K_0$ , which enhances the rate of matter production — by 18 orders of magnitude.

The scalar-cosmic expansion, on the other hand, is govern by the equations

$$\begin{aligned}\frac{dH}{dt} &= \frac{k}{a^2} - 2 \left( \frac{dF}{dt} \right)^2 + \frac{a}{3} \frac{\partial V}{\partial a} - \frac{1}{a^3} (E_m + E_v) \\ \frac{d^2 F}{dt^2} &= -3H \frac{dF}{dt} - \frac{\zeta_0 E_v}{a^3} F - \frac{1}{2} \frac{\partial V}{\partial F}\end{aligned}\tag{33}$$

where  $H = a^{-1} da/dt$ , with the constraint

$$H^2 + \frac{k}{a^2} - \frac{2}{3} \left( \dot{F}^2 + V + \frac{1 + \zeta_0}{a^3} E_v + \frac{1}{a^3} E_m \right) = 0\tag{34}$$

In these equations,  $E_m, E_v$  are practically constants. The solutions are qualitatively the same as those for the real scalar field described in I, with asymptotic behavior  $H \sim t^{-p}$ .

To summarize:

- From the point of view of the cosmic expansion, the vortex-matter system is essentially static.
- The cosmic expansion is extremely fast from the viewpoint vortex-matter system, but it is noticeable only as an "abnormally" large rate of matter production.

## VI. THE INFLATION ERA

The inflation scenario is designed to explain the presently observed large-scale uniformity of galactic distribution in the universe. It assumes that all matter was created when the universe was so small that they stay within each other's event horizon, and so maintain a uniform density. The era comes to an end when, with the expansion of the universe, its size is inflated to such an extend that the matter fall out of each other's event horizon, but they retain the memory of a uniform density. Traditional estimates puts the inflation factor at some 27 orders of magnitude [8]. In our model, we picture matter to be created in the vortex tangle, which has a finite lifetime, and this lifetime is the duration of the inflation

era. For the scenario to work, matter creation must be mostly complete by the time the vortex tangle decays.

To illustrate the model with definite numbers, we shall make some phenomenological assumptions about the parameters  $\gamma$  and  $K_0$  in (32). We assume that  $K_0$  is a large constant, and

$$\gamma = \frac{A}{1 + B\tau} \quad (35)$$

where  $A$  and  $B$  are constants. This embodies the physical reason behind the demise of the vortex tangle, namely, the cosmic expansion reduces the "head wind" necessary for its sustenance. Without detailed computations, we can see that the qualitative behaviors are as shown in Fig.1. The vortex energy rises through a maximum and decays with a long tail, like  $\tau^{-1}$ . The characteristic time  $\tau_0$  defines the lifetime of the tangle, and therefore that of the inflation era. The total matter energy  $E_m$  is proportional to the area under the curve for  $E_v$ . It approaches a constant  $E_0$ , which is the total energy of matter created during the inflation era.

We can now put in some numbers. The lifetime of the tangle  $\tau_0$  corresponds to the Planck time  $t_0 = \tau_0/s_1$ . According to the power-law obtained in I, the radius of the universe expands by a factor  $a(t_0)/a_0 = \exp(\xi t_0^{1-p})$ . With  $s_1 \sim 10^{-18}$ , and taking  $\tau_0 \sim 1$ ,  $\xi = 1$ ,  $p = 0.9$ , we obtain

$$t_0 \sim 10^{18} (10^{-26}\text{s}) \quad (36)$$

$$\frac{a(t_0)}{a_0} \sim 10^{27} \quad (37)$$

We can easily adjust  $K_0$  to yield whatever fraction of the total energy in the universe:

$$E_0 \approx 10^{22} m_{\text{sun}} = 2 \times 10^{69} \text{joule} \quad (38)$$

Our picture of the inflation era is completely different from the conventional scenario [8]. In the latter, the scalar field starts at zero field, at a "false vacuum" corresponding to a potential maximum, and does a "slow roll" to the potential minimum, the "true vacuum". The field then oscillates about the minimum, and creates matter through "reheating". In our model, however, there was no "slow roll", as we learn in I. Instead, the field performs rapid oscillations with very large amplitudes that sample the asymptotic exponential region of the HH potential. During this era of rapid oscillation, a vortex tangle rises and falls, and all matter was created through the vortex reconnections essential to the tangle's maintenance.

There is suggestion that a vortex tangle can arise in string theory [11].

## VII. THE POST-INFLATION UNIVERSE

After the inflation era, the standard hot big bang scenario takes over. In this regime, spatial non-uniformity becomes the interesting feature, and our model ceases to be valid. It paved the way for hot big bang theory through decoupling of nucleogenesis and galaxy formation from cosmic expansion. The model, however, does have an important legacy: it leaves the universe remains in a superfluid state, leading to observable manifestations, as discussed in the following.

### A. Galactic voids

After the demise of the vortex tangle, there will be leftover vortex tubes. These tubes are devoid of the scalar field, and presumably no matter was ever created inside. Their cores must expand with the universe, and by would have grown to the enormous voids observed in the distribution of galaxies. Matter created outside the vortex tubes tend to accumulate at the tube surface, due to a lowering of the hydrodynamic pressure caused by a higher tangential superfluid velocity there. In superfluid liquid helium, this effect has been demonstrated, through the coating of vortex tubes by dissolved metallic nanoparticles [12]. In Fig.2 we simulate galactic voids arising from three vortex tubes, with comparison to the observed "stick man" configuration [13].

### B. Varieties of vortices

The galactic voids corresponds to vortex tubes in the primordial scalar field, which were created right after the big bang. Different types of vacuum field could emerge with the creation of matter, giving rise to different types of superfluids with their own vortices, of different core sizes. The possible types of vacuum fields would be determined by particle theory.

In the cosmological context, the presence of different types of superfluids could be likened to a mixture of liquid  $^4\text{He}$  and  $^3\text{He}$  at temperatures below  $10^{-3}\text{K}$ , when both are superfluids.

In such a mixture, The core of a vortex tube could be devoid of  $^4\text{He}$  but not  $^3\text{He}$ , and vice versa, or it could be devoid of both. Added to the complexity is the fact that the  $^4\text{He}$ - $^3\text{He}$  mix can exist in various phases, depending on the temperature and the relative concentration, in which the two liquids either commingle or segregate. In the cosmological context, the coexistence of a variety of superfluids would present rich phenomena, on which we are not in a position to speculate.

When we refer to "the superfluid" or "the vortex tube" in the following, we do not commit ourselves to a specific type, but merely suggest generic behaviors.

### C. Dark mass

Given that the universe is filled with superfluids of various kinds, all galaxies should move through them without friction, as long as their velocities lie below critical values. However, it has been shown theoretically that a superfluid can be pinned to a random potential [14]. A galaxy could be perceived as a random potential by an underlying superfluid, due to randomness in the stellar distribution. Thus, a galaxy could drag the superfluid along in its rotation, and acquire extra moment of inertia. This would be perceived by us as dark mass. The superfluid dragged along in galactic rotation would be separated from the stationary background fluid by a boundary layer laced with vortex lines, and these could also contribute to the dark mass [15]. There have been others works on a superfluid origin of the dark mass [16].

### D. Non-thermal filaments

The superfluid could be pinned to not only a galaxy, but any congregation of matter that it perceived as a random potential, and that may include rotating star clusters within a galaxy. As mentioned above, such rotating stellar objects, being it galaxy or star cluster, would be encaged in vortex lines. The cores of some types of vortex lines could trap matter and shine. In liquid helium, vortex cores have been made visible through the trapping of hydrogen ice [17]. In the astrophysical context, such vortex lines could be candidates for the "non-thermal filaments" observed near the center of the Milky Way [18], as illustrated in Fig.3.



## E. Jet events

Vortex lines in the later universe will be sparsely distributed, compared to those in vortex tangle of the early universe; but once in a while they could find each other and reconnect. As discuss in Appendix B, the signature of a reconnection is the production of two jets of energy. This could be the mechanism behind the observed gamma ray bursts and cosmic jets.

## ACKNOWLEDGMENTS

One of us (K.H.) has benefited by conversations with D.P. Lathrop on quantum turbulence and possible cosmological applications. In particular he thanks him for directing attention to the “non-thermal filaments”.

## Appendix A: Static vortex solution

We solve for a static vortex solution to the complex scalar field equation in flat space-time:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\phi - \frac{\partial V}{\partial \phi^*} = 0 \quad (\text{A1})$$

where  $V$  is the Halpern-Huang potential. The equations of motion in the phase representation  $\phi = Fe^{i\sigma}$  are

$$\begin{aligned} \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)F + F\dot{\sigma}^2 - F|\nabla\sigma|^2 - \frac{\partial V}{\partial F} &= 0 \\ \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\sigma - \frac{2}{F}\frac{\partial F}{\partial t}\frac{\partial \sigma}{\partial t} + \frac{2}{F}\nabla F \cdot \nabla\sigma &= 0 \end{aligned} \quad (\text{A2})$$

Consider an infinite vortex line along the  $z$ -axis with unit quantized vorticity, such that

$$\oint_C \nabla\sigma \cdot d\mathbf{s} = 2\pi \quad (\text{A3})$$

where  $C$  is a circle about the origin in the  $xy$  plane. This gives  $\sigma = \theta$ , in cylindrical coordinates  $(r, \theta)$ . Thus  $\nabla\sigma = \hat{\theta}r^{-1}$ ,  $|\nabla\sigma|^2 = r^{-2}$ , and the equation for  $F$  becomes

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} - \frac{F}{r^2} - \frac{\partial V}{\partial F} = 0 \quad (\text{A4})$$

Putting

$$\begin{aligned} F &= \frac{f}{a} \\ r &= \frac{\rho}{a} \end{aligned} \tag{A5}$$

we have

$$f'' + \frac{f'}{\rho} - \frac{f}{\rho^2} - \frac{\partial V}{\partial f} = 0 \tag{A6}$$

where for the HH potential  $V$  we have

$$\frac{\partial V}{\partial f} = -f M(-1 + b/2, 2, 16\pi^2 f^2) \tag{A7}$$

where  $M$  is the Kummer function. The boundary conditions are

$$\begin{aligned} f(0) &= 0 \\ f(\infty) &= \text{Nonzero constant} \end{aligned} \tag{A8}$$

We take  $b = 1.5$ , and find the numerical solution by "shooting", i.e., adjusting the initial conditions so as to get a nonzero  $f(\infty)$ . We obtain the desired behavior with  $f(0) = 0.001$ ,  $f'(0) = 0.2559$ . The field modulus  $f(\rho)$  is plotted in Fig.4. The high-energy cutoff  $\Lambda$  suppresses the field at small distances, with the functional form of the field dependent on the cutoff function. We simply set

$$F(r) = \begin{cases} F(\infty) & (r > R_0) \\ 0 & (r < R_0) \end{cases} \tag{A9}$$

where  $R_0 \sim \Lambda^{-1}$  is the core radius. With this approximation, the scalar field is uniform in a multiply-connected space.

## Appendix B: Vortex dynamics

A simple vortex structure is the vortex ring, whose vortex line is a directed circle of radius  $R$ , as illustrated in Fig.5. The ring moves normal to its own plane, in a direction in accordance with the right-hand rule, with velocity [19]

$$v = \frac{1}{4\pi R} \ln \frac{R}{R_0} \tag{B1}$$

where  $R_0$  is proportional to the core radius. The logarithmic factor  $\ln(R/R_0)$  is slowly-varying, and may be regarded as a constant for all practical purposes. Thus  $v \propto R^{-1}$  approximately. We can qualitatively understand the motion of an arbitrary vortex line as follows. At any point on the vortex line there is a radius of curvature  $R$ , which we can associate with an imaginary vortex ring of the same radius, tangent to the line at that point. The local translational velocity would be  $v \propto R^{-1}$  normal to this ring. The more sharply a vortex line bends, the faster it moves perpendicular to the bending. In this manner, a vortex line generally executes complicated self-induced motion, as illustrated in Fig.5. The local velocity  $v(s)$  of the vortex line, where  $s$  is a parameter along the vortex line, is also the velocity of the superfluid at that point.

The reconnection of vortex lines proposed by Feynman [6] is illustrated in fig.6. It has been simulated via the nonlinear Schrödinger equation [20]. This mechanism is important for the formation of the vortex tangle, in the following scenario according to Schwarz [21,22]. Vortex rings will grow when there is a normal fluid head wind, i.e., counter heat flow opposed to the ring's translational motion, and shrink in a tail wind. Given a distribution of vortex rings, some will grow to large sizes, and inevitably reconnect, as schematically illustrated in Fig.7. The reconnection produces a set of smaller rings, some of which will again grow and reconnect, and so forth, until there is vortex tangle, like the one shown in Fig.7 through computer simulation, with a fractal dimension 1.6 [23]. The steady-state of a vortex tangle is maintained by a constant rate of growth and reconnections. If the heat source is removed, the vortex tangle will decay into a sparse collection of contracting vortex rings, and eventually disappear into the sea of quantum fluctuations [24].

Reconnection occurs between two antiparallel vortex lines. Computer simulation shows that parallel vortex lines tend to reorient themselves at close approach in order to reconnect [22]. The critical distance for reconnection between two vortex lines with the same radius of curvature  $R$  is given by [22]

$$\delta \approx 2R \ln \frac{R}{c_0 R_0} \quad (\text{B2})$$

where  $c_0$  is a constant. Here, the logarithmic factor is practically a constant. Comparison with (B1) shows  $\delta \propto v^{-1}$ , where  $v$  is the relative velocity of the vortex segments.

As illustrated in Fig.7, reconnection creates two cusps on the newly constituted vortex lines, with very small radii of curvature. Consequently, the cusps will spring away from each other at very high speed, creating two oppositely directed jets of energy, which are signature

events of vortex reconnection.

In Vinen's equation  $\dot{\ell} = A\ell^{3/2} - B\ell^2$ , the coefficients  $A$  and  $B$  should embody all the effects discussed above. In liquid helium,  $A$  is proportional to the speed of the normal fluid. This equation has also been derived from vortex dynamics, and  $A$  and  $B$  can be expressed in terms of properties of the system of vortex lines [22]. However, they do not always agree with the phenomenological view.

In superfluid helium, experiments reveal that the velocity distribution in the tangle deviates from that in classical turbulence, in that it has a fat non-Gaussian tail [17]. Reconnection events have been observed and studied statistically [25].

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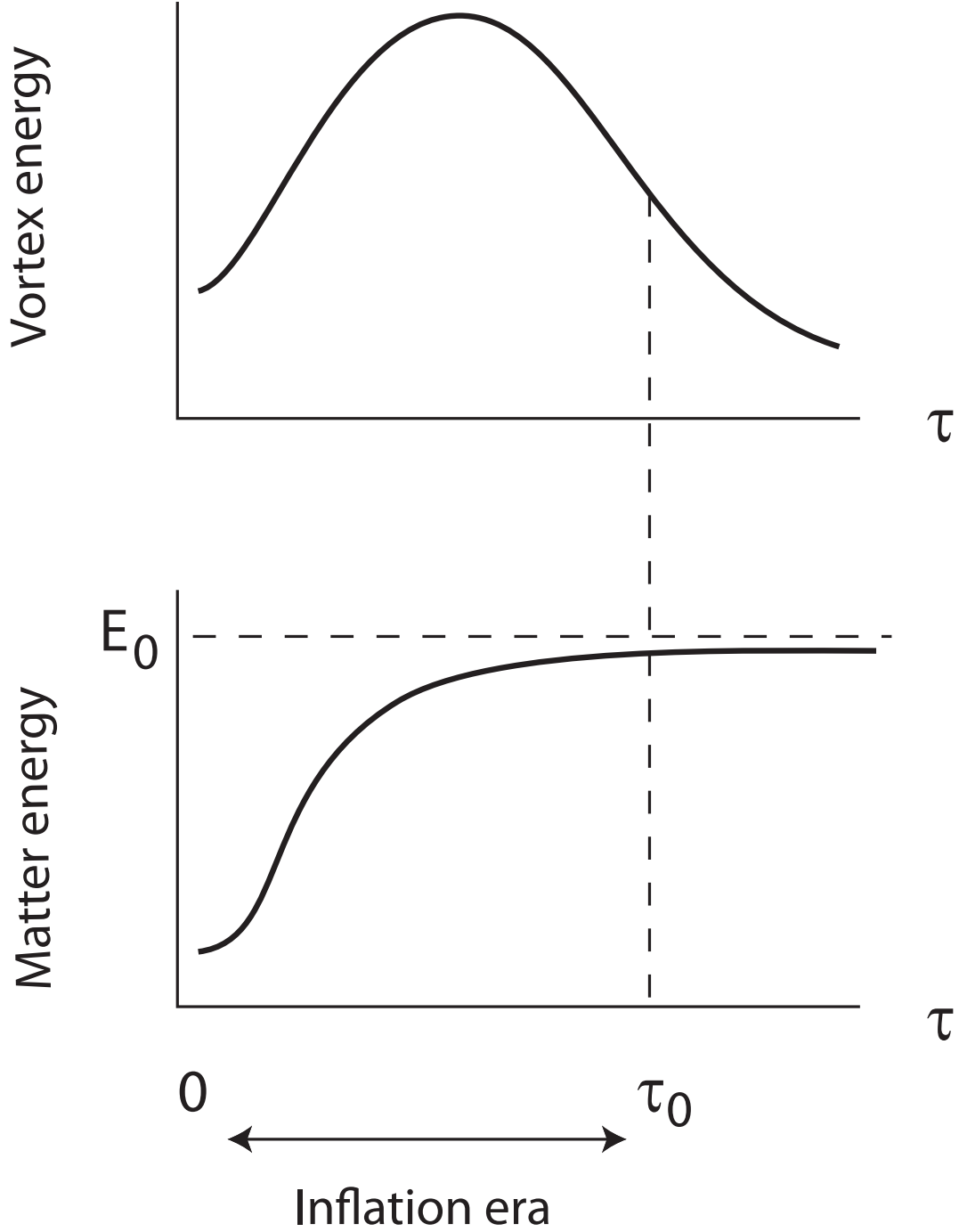


FIG. 1. Upper panel shows total energy of the vortex tangle (quantum turbulence) as function of nuclear time  $\tau$ , which is related to the Planck time  $t$  by  $\tau = s_1 t$ , with  $s_1 \sim 10^{-18}$ . The lifetime  $\tau_0$  of the vortex tangle is the duration of the inflation era, which can be estimated to be  $10^{-26}$ s. By the same estimate, the radius of the universe increased by a factor  $10^{27}$ . Lower panel show total energy of matter produced, which is proportional to the area under the curve in the upper panel. The total energy  $E_0$  can be adjusted to correspond to the total observed energy in the universe, the order of  $10^{22}$  solar masses.

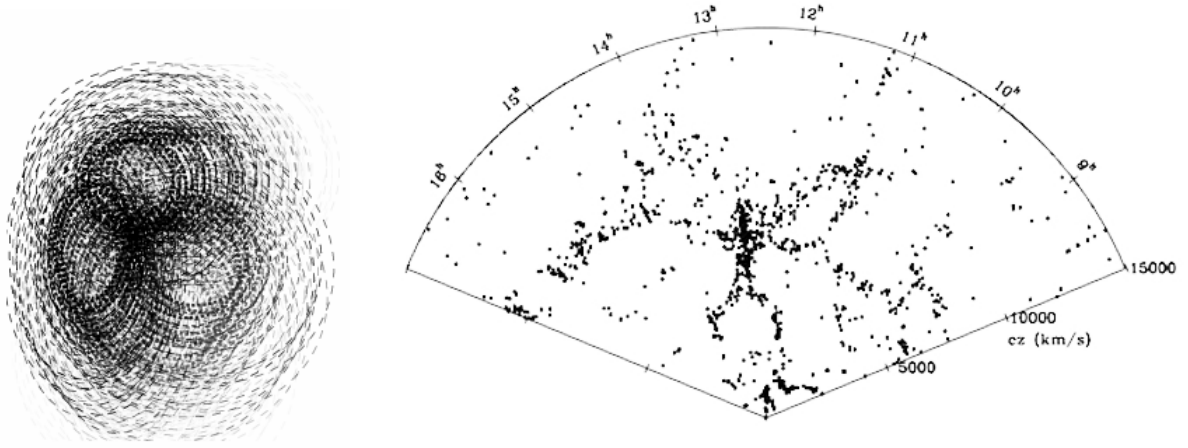


FIG. 2. Left panel: Simulation of galactic voids by superposition of three vortex tubes, whose cores, originally of Planck scale near the big bang, have grown with the expanding universe, and reached hundreds of million of light years, in the 15 billion years since. The vortex cores are devoid of the vacuum scalar field, and therefore of matter. Galaxies formed outside adhere to tube surfaces due to hydrodynamic pressure. Right panel: The "stickman" configuration observed in galactic distributions, from Ref.[12].



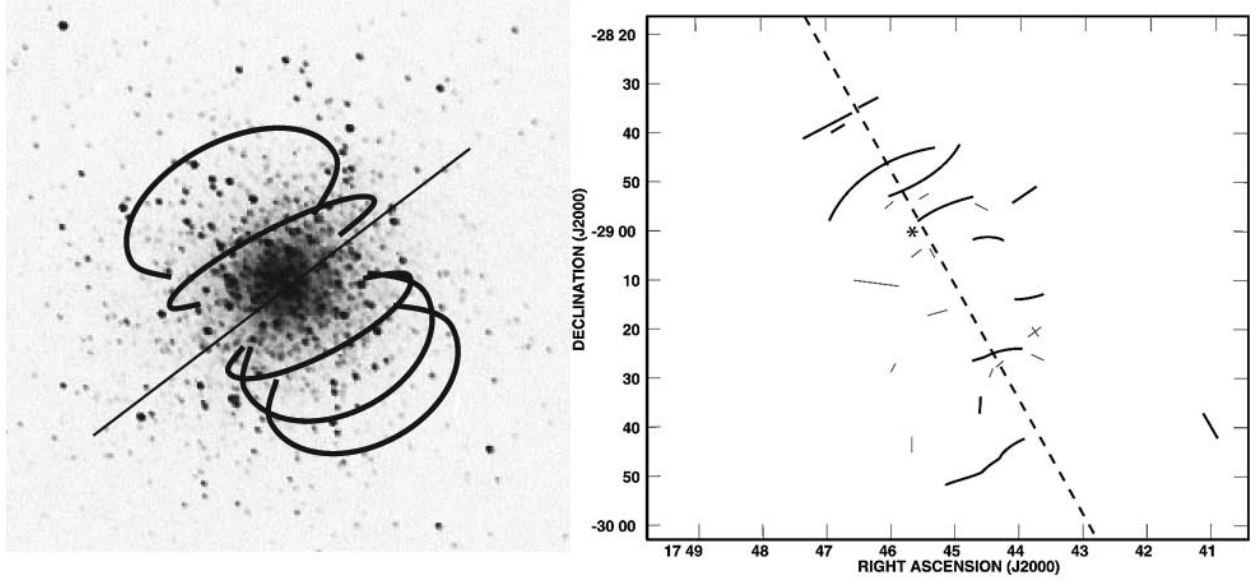


FIG. 3. Left panel shows a drawing of a rotating stellar distribution, which could drag along the cosmic superfluid it is immersed in, if it has sufficient randomness. The stellar system will then acquire extra moment of inertia, perceived by us as "dark mass". The co-moving superfluid will be separated from the stationary background fluid by a boundary layer that is laced with vortex tubes. These could be the "non-thermal filaments" observed near the center of the Milky Way, a schematic drawing of which, from Ref.[15], is reproduced in the right panel.

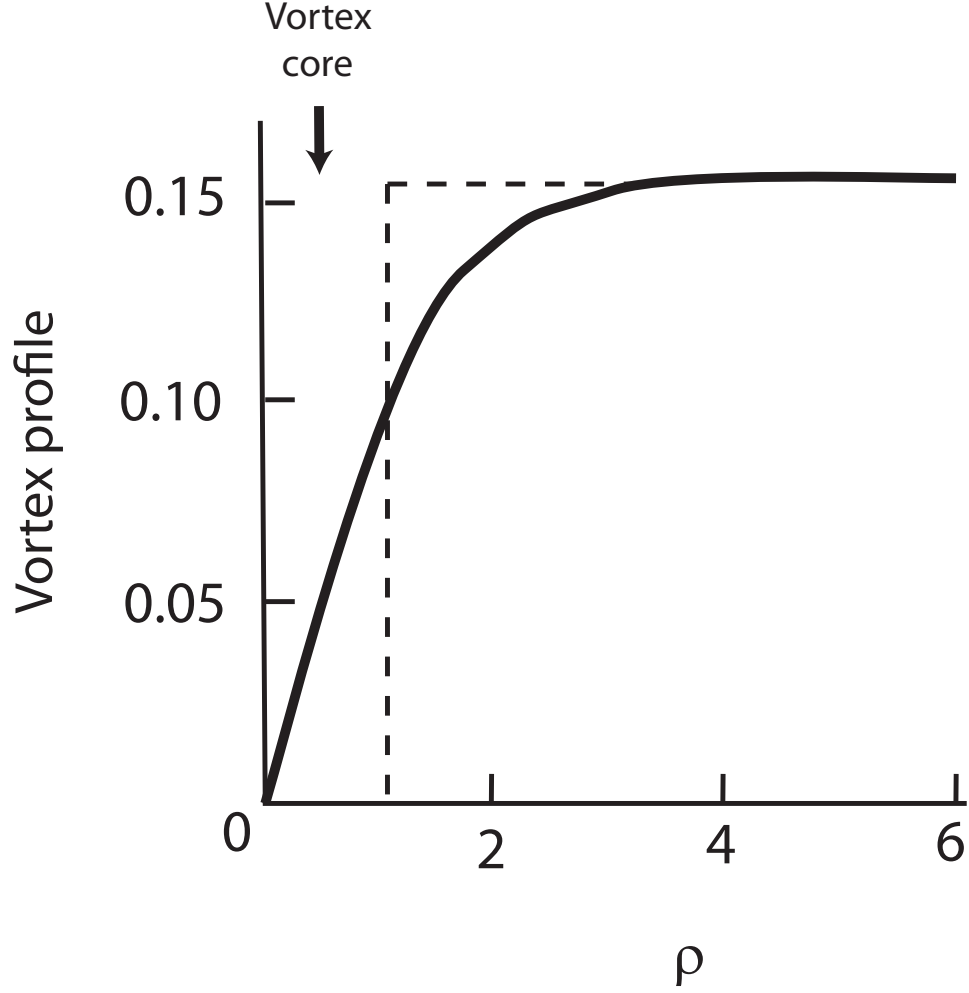


FIG. 4. Profile of field modulus in a vortex solution with infinite vortex line along the  $z$ -axis, as a function of reduced distance  $\rho$  from the line. The field near  $\rho = 0$  is further suppressed by the short-distance cutoff, and this creates a vortex core. We approximate the configuration with a sharp cutoff, so the field outside the core is constant.

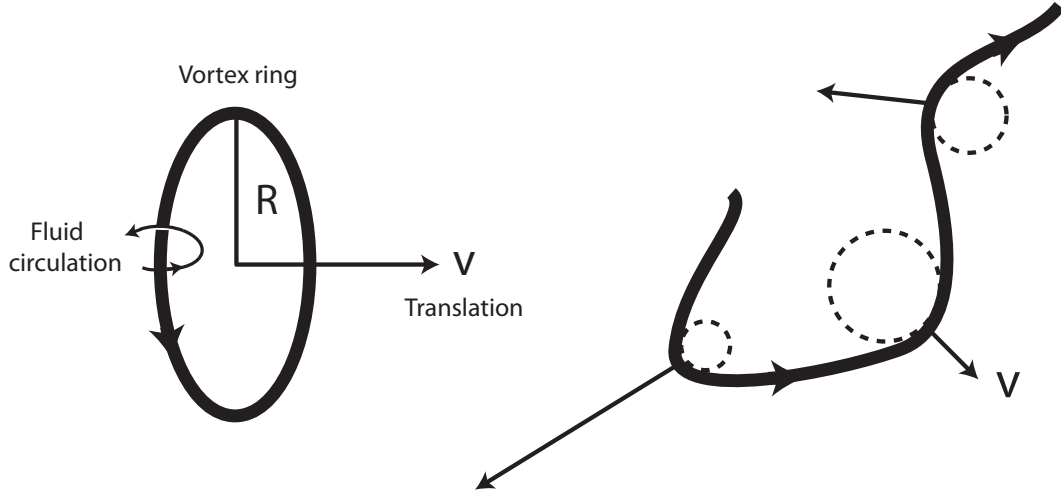


FIG. 5. The heavy lines in these picture denote the vortex core, which has a direction specified by the vorticity. The vortex ring moves in a direction consistent with the right-hand rule, with a velocity approximately inversely proportional to it radius. A vortex tube moves in such a fashion such that the local velocity at any point is that of a tangential vortex ring with the local radius of curvature.

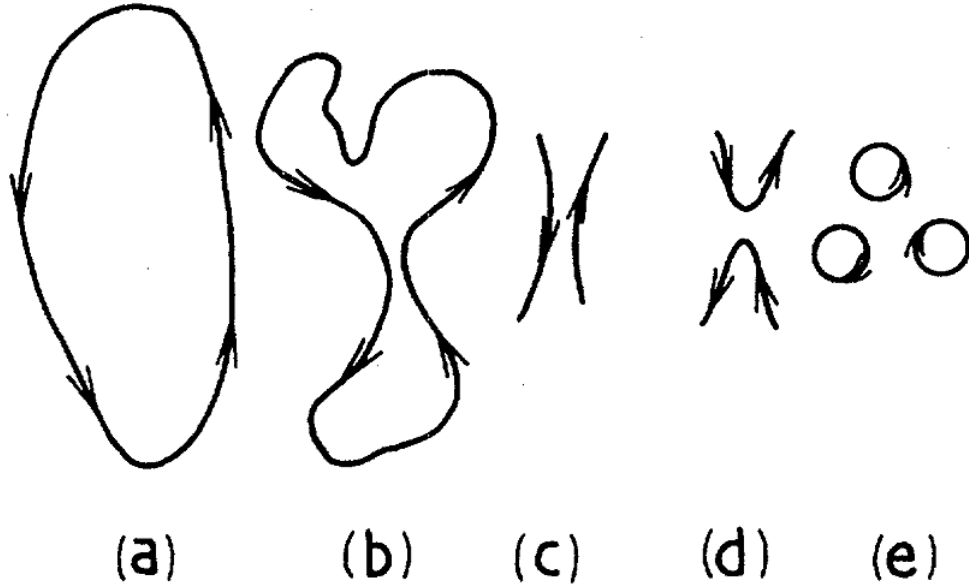


FIG. 6. Feynman's sketch of the decay of a quantized vortex ring from Ref.[6]. Through reconnections, a large vortex ring become smaller rings, and smaller rings become even smaller ones, and so on, to quantum turbulence.

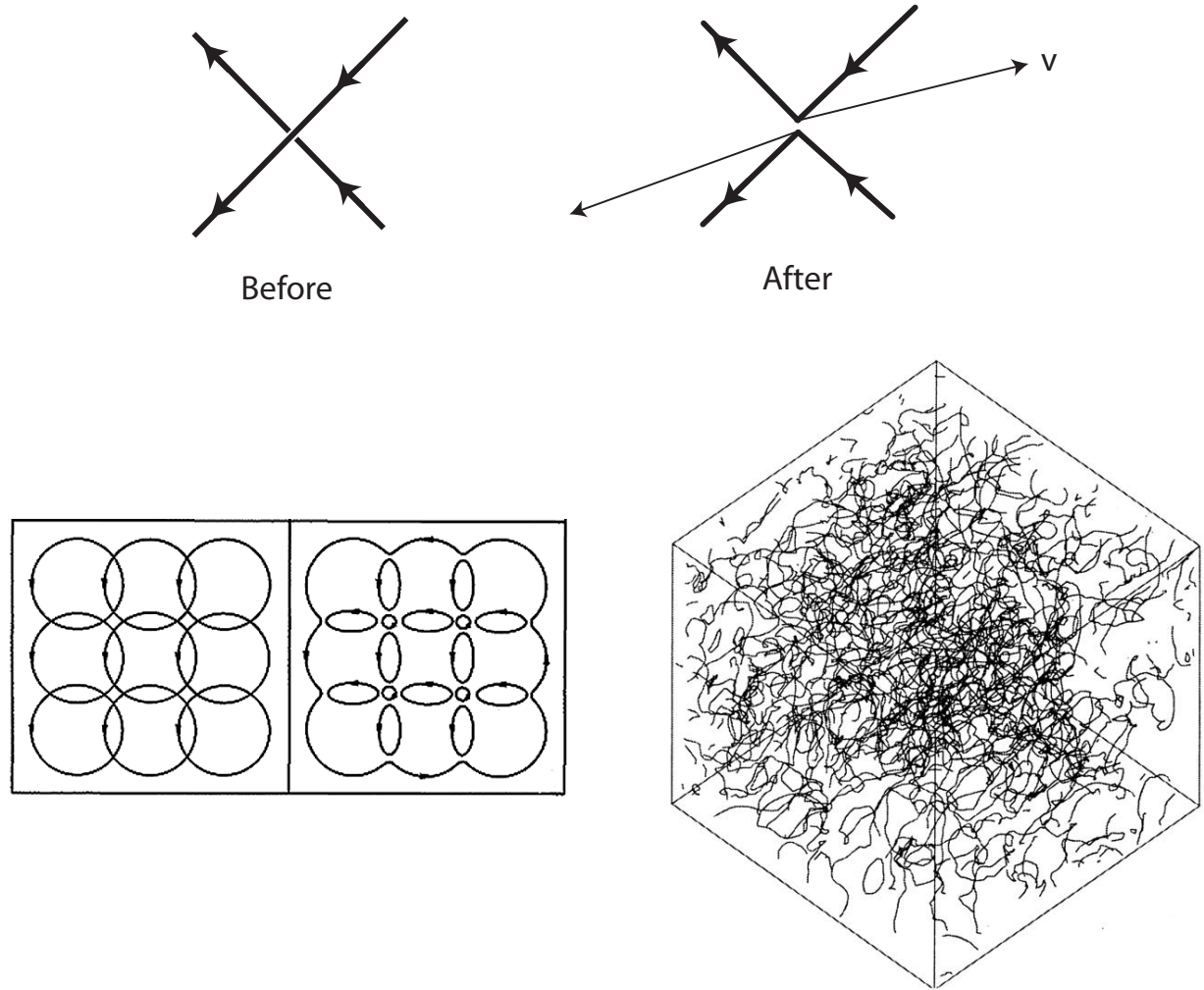


FIG. 7. Upper panel: Immediately after reconnection, two cusps occur on the participating vortex lines, which, because of the near-zero radii of curvature, spring away from each other with theoretically infinite speed, creating two jets of energy. Lower panel: Left side schematically illustrates emulsification of system of vortex rings due to reconnections, from Ref.[18]. Right side show a fully-formed vortex tangle of fractal dimension 1.6., from Ref.[17].